University of the People

MATH 1211 Calculus 1

Unit 4 Written Assignment

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1. Chains Inc. is in the business of making and selling chains. Let **[ c(t) ](https://my.uopeople.edu/filter/tex/displaytex.php?texexp=%20c%28t%29%20)**be the number of miles of chain produced after **[ t ](https://my.uopeople.edu/filter/tex/displaytex.php?texexp=%20t%20)** hours of production. Let **[ p(c) ](https://my.uopeople.edu/filter/tex/displaytex.php?texexp=%20p%28c%29%20)** be the profit as a function of the number of miles of chain produced and let **[ q(t) ](https://my.uopeople.edu/filter/tex/displaytex.php?texexp=%20q%28t%29%20)** be the profit as a function of the number of hours of production. Suppose the company can produce 3 miles of chain per hour and suppose their profit on the chains is $4000 per mile of chain.  Find and interpret (use complete sentences) each of the following (include units), **[ c'(t) ](https://my.uopeople.edu/filter/tex/displaytex.php?texexp=%20c%27%28t%29%20)**, **[ p'(c) ](https://my.uopeople.edu/filter/tex/displaytex.php?texexp=%20p%27%28c%29%20)**, and **[ q'(t) ](https://my.uopeople.edu/filter/tex/displaytex.php?texexp=%20q%27%28t%29%20)**.  How does **[ q'(t) ](https://my.uopeople.edu/filter/tex/displaytex.php?texexp=%20q%27%28t%29%20)** relates to **[ p'(c) ](https://my.uopeople.edu/filter/tex/displaytex.php?texexp=%20p%27%28c%29%20)**and **[ c'(t) ](https://my.uopeople.edu/filter/tex/displaytex.php?texexp=%20c%27%28t%29%20)**? 

C’(t) is the derivative of number of miles of chain produced after t hour of production. It’s the instantaneous rate of production of chain by the unit of miles/hour.

P’(c) is the derivative of profit function over the speed of chain production by the uinit of dollar/miles.

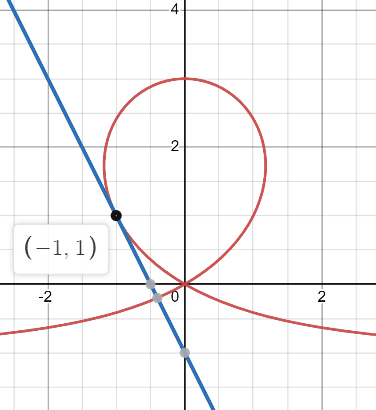
q’(t) is the derivative of profict function over the number of hour production.

Its indicating the hourly profit by the unit of dollar/hour.

Then the q’(t) is chained rule product of p’(c) and c’(t)

q’(t) = c’(t)q’(c)

2. Use Desmos to graph the function **[ y^3+yx^2+x^2-3y^2=0 ](https://my.uopeople.edu/filter/tex/displaytex.php?texexp=%20y%5E3%2Byx%5E2%2Bx%5E2-3y%5E2%3D0%20)** and estimate the slope of the tangent line at (-1,1). Then find **[ \frac{dy}{dx} ](https://my.uopeople.edu/filter/tex/displaytex.php?texexp=%20%5Cfrac%7Bdy%7D%7Bdx%7D%20)** using implicit differentiation and plug in **[ x=-1 ](https://my.uopeople.edu/filter/tex/displaytex.php?texexp=%20x%3D-1%20)** and **[ y=1 ](https://my.uopeople.edu/filter/tex/displaytex.php?texexp=%20y%3D1%20)**. Compare and discuss the estimated slope with the slope you found analytically.



Start by differentiating,

3y2 +x2+2xy+2x-6y=0

(3y2+x2 -6y)=-2x-2xy

= = (-2(-1) -2(-1)) /(3+1-6) = 4/-2=-2

Plug in to point slope

y-1 = -2 (x+1)

y=-2x -1

by ploting the graph, we noticed that the tangent line at (-1,1) looks close to what we analyse that the tangent line from the differentiation.

3. Let **[ f(x)=(3x^2+1)^2 ](https://my.uopeople.edu/filter/tex/displaytex.php?texexp=%20f%28x%29%3D%283x%5E2%2B1%29%5E2%20)**. Find **[ f'(x) ](https://my.uopeople.edu/filter/tex/displaytex.php?texexp=%20f%27%28x%29%20)** in 3 different ways by following the instructions below in parts a, b and c:

a) Develop the identity **[ f(x) ](https://my.uopeople.edu/filter/tex/displaytex.php?texexp=%20f%28x%29%20)** then take the derivative.

F(x)= 9x4+6x2+1

F’(x) = 4\*9x3+12x+0 = 36x3+12x

b) View **[ f(x) ](https://my.uopeople.edu/filter/tex/displaytex.php?texexp=%20f%28x%29%20)** as **[ (3x^2+1)(3x^2+1) ](https://my.uopeople.edu/filter/tex/displaytex.php?texexp=%20%283x%5E2%2B1%29%283x%5E2%2B1%29%20)** and use the product rule to find **[ f'(x) ](https://my.uopeople.edu/filter/tex/displaytex.php?texexp=%20f%27%28x%29%20)**.

According to product rule,

d(3x2+1)/dx = 6x

F’(x) = 6x\*(3x2+1) + 6x\*(3x2+1) =36x3 + 12x

c) Apply the chain rule directly to the expression **[ f(x)=(3x^2+1)^2 ](https://my.uopeople.edu/filter/tex/displaytex.php?texexp=%20f%28x%29%3D%283x%5E2%2B1%29%5E2%20)**.

Let 3x2+1 = u

F’(x) = 2(3x2+1) \* 6x = (6x2+2) 6x =36x3 + 12x

d) Are your answers in parts a, b, c the same? Why or why not?

The answers from a,b and c are the same.

The reason is that thay are all by the same mathematical definition that indicates

The instantaneous rate of change of the function.

4. Find **[ \frac{dy}{dx} ](https://my.uopeople.edu/filter/tex/displaytex.php?texexp=%20%5Cfrac%7Bdy%7D%7Bdx%7D%20)** for the equation **[ 3y^2-cosy=x^3 ](https://my.uopeople.edu/filter/tex/displaytex.php?texexp=%203y%5E2-cosy%3Dx%5E3%20)**

Using chain rule and implicit differentiation,

Taking derivative of both side,

6y +siny = 3x2

(6y+siny)=3x2

= 3x2/(6y+siny)

5. Find the equation of the tangent line that passes through point (1,2) to the graph **[ 8y^3+x^2y-x=65 ](https://my.uopeople.edu/filter/tex/displaytex.php?texexp=%208y%5E3%2Bx%5E2y-x%3D65%20)**  
By applying the differentiation rule,

24y2 +2xy+x2-1 =0

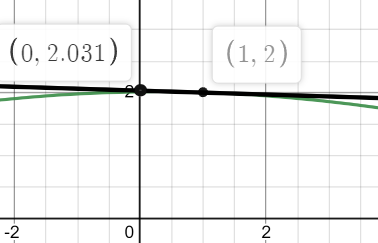
(24y2 +x2) = 1-2xy

Point slope plugin,

y-2=-3/97(x-1)

y= -3x/97 +3/97 + 2

y= -3x/97+ 197/97



6. Find **[ \frac{dy}{dx} ](https://my.uopeople.edu/filter/tex/displaytex.php?texexp=%20%5Cfrac%7Bdy%7D%7Bdx%7D%20)** for the equation **[ xy^2-x^2y=2
    ](https://my.uopeople.edu/filter/tex/displaytex.php?texexp=%20xy%5E2-x%5E2y%3D2%0D%0A%20%20%20%20)**

Applying the differentiating rule,

Y2+ 2xy – (2xy + x2 ) = 0

Y2 +(2xy-x2)-2xy = 0

7. Find **[ f'(x) ](https://my.uopeople.edu/filter/tex/displaytex.php?texexp=%20f%27%28x%29%20)** for the function **[ f(x)= \sqrt[4]{-3x^4-2} ](https://my.uopeople.edu/filter/tex/displaytex.php?texexp=%20f%28x%29%3D%20%5Csqrt%5B4%5D%7B-3x%5E4-2%7D%20)**

-3x4-2 is negative and the domain is not exist thus the derivative is not exist also.

So the tangent line did not exisit as well. Thus f’(x) is not existing.

8. Find **[ f'(x) ](https://my.uopeople.edu/filter/tex/displaytex.php?texexp=%20f%27%28x%29%20)** for the function **[ f(x)=(5x^2+3)^4 ](https://my.uopeople.edu/filter/tex/displaytex.php?texexp=%20f%28x%29%3D%285x%5E2%2B3%29%5E4%20)**

Using chain rule,

F’(x)=4(5x2+3)3\*(10x) = 40x(5x2+3)3

9. Find **[ f'(x) ](https://my.uopeople.edu/filter/tex/displaytex.php?texexp=%20f%27%28x%29%20)** for the function  **[ f(x)=sin^2(cos(4x)) ](https://my.uopeople.edu/filter/tex/displaytex.php?texexp=%20f%28x%29%3Dsin%5E2%28cos%284x%29%29%20)**

Using chain rule twice,

F’(x) = 2(sin(cos(4x)))\* cos(cos(4x)) \* -sin(4x) \* 4

10. Find **[ f'(x) ](https://my.uopeople.edu/filter/tex/displaytex.php?texexp=%20f%27%28x%29%20)** for the following functions:

a) **[ f(x) = cos(ln4x^3))](https://my.uopeople.edu/filter/tex/displaytex.php?texexp=%20f%28x%29%20%3D%20cos%28ln4x%5E3%29%29)**

b) **[ f(x)= e^{(4x^3+5)^2} ](https://my.uopeople.edu/filter/tex/displaytex.php?texexp=%20f%28x%29%3D%20e%5E%7B%284x%5E3%2B5%29%5E2%7D%20)**

1. F’(x) = -sin(ln4x3) \* =

According to special derivative rule,

1. F’(x) =e(4x^3+5)^2 \* 2(4x3+5)\* 12x2 = 24x2(4x3+5) e(4x^3+5)^2